Abstract

This paper proposes a transmission ratio scheduling and control methodology for a vehicle with a Continuous Variable Transmission (CVT) and a downsized gasoline engine. The methodology is designed to deliver the optimal vehicle fuel economy within drivability and performance constraints. Traditionally, the Optimum Operating Line (OOL) generated from an engine brake specific fuel consumption map is considered to be the best option for ratio scheduling, as it defines the points at which engine efficiency is maximized. But the OOL does not consider transmission efficiency, which may be a source of significant losses. To develop a CVT ratio schedule that offers the best fuel economy for the complete powertrain, an empirical approach was used to minimize fuel consumption by considering engine efficiency, CVT efficiency, and requested vehicle power. A backward-looking model was used to simulate a standard driving cycle (FTP-75) and develop a new powertrain-optimal operating line (P-OOL). Simulation results using the backward-looking model show a significant improvement in overall fuel economy when using the P-OOL (considers engine and CVT efficiency) compared to the OOL (considers only engine efficiency). Next, a forward-looking, velocity-driven model was developed to simulate the real-time behavior of a vehicle. Fuel economy results were compared when implementing the P-OOL and the OOL with a hardware-based CVT shift rate constraint. Finally, a control algorithm that considers powertrain loss and inertia torque due to CVT ratio changes is proposed to minimize powertrain response lag when operating along the P-OOL. This combined ratio scheduling and response lag control methodology is shown to improve vehicle fuel economy with real-time simulated driving conditions.

Introduction

In the downsized vehicles segment, the Continuous Variable Transmission (CVT) has emerged as an efficient alternative to stepped ratio gearbox transmissions. The infinite range of gear ratio availability in a CVT enables the engine to operate at the most efficient operating points for longer periods of time. One type of CVT often used in automotive applications is the variable diameter CVT. This CVT is a belt and pulley system, where the two pulleys are each made of two conical sheaves facing each other. The primary pulley is connected to the engine crankshaft, usually with a torque converter or centrifugal clutch. The secondary pulley is attached to the drive shaft and transmits power to the wheels through a final gear reduction. For both pulleys, the sheaves are allowed to move closer and farther apart. Since the width of the belt running between them remains constant, this movement changes the diameter of the pulleys, allowing the CVT ratio (the ratio of primary pulley speed to secondary pulley speed) to change. This allows the CVT to operate at gear ratios between the stepped ratios at which a gearbox transmission can operate, ultimately allowing the engine to consume less fuel for the same power output.

Both the engine and CVT efficiencies have a direct impact on the fuel economy. It is known that engine efficiency is highest when the engine operates at points along the optimal operating line (OOL) [1]. The OOL is defined as the line of least specific fuel consumption across the operating speed and torque range of the engine. As shown in Figure 1, it is created by connecting the points where each engine power contour intersects the highest engine efficiency contour.

Several studies have investigated powertrain control under the assumption that OOL tracking yields the highest fuel economy. These have demonstrated the advantages of CVT-implemented systems over stepped-ratio transmissions [2], demonstrated the effects of a multivariable control structure on powertrain performance [3], and considered fuel-optimal control strategies during transient operation [4]. However, the fuel economy of a vehicle is dependent on the transmission efficiency as well as the engine efficiency. Operating the engine along the OOL neglects the effect of CVT efficiency on fuel economy. Past studies have considered CVT efficiency through the use of analytical equations [5,6]. These methods are effective in simulation, but are system-specific and may require long processing times. In the methodology
presented in this paper, engine and CVT efficiencies, as well as fuel consumption data, are approximated and interpolated from empirical data sets. The methods for approximating the data were developed for faster and smoother optimization, thereby reducing computational load and processing time.

CVT powertrain response lag time is also an important consideration in fuel economy optimization. Powertrain lag results from a number of factors including CVT shift dynamics and CVT filling time - the time it takes for the hydraulic fluid to engage the clutches that move the sheaves of the CVT pulleys. This effect has been considered in control algorithm development [7,8] using a velocity-based approach for development of the optimal control. Here, a similar approach is taken and extended to the selection of operation points to improve the overall CVT efficiency.

The first approach presented here is a backward-looking, cycle-driven method to optimize CVT ratio scheduling while incorporating varying engine and CVT efficiency. The optimization result obtained from the backward-looking model is used to generate a new powertrain-optimal operating line (P-OOL), which considers both engine and CVT efficiency. We investigate the improvement in fuel economy by tracking the P-OOL in the backward-looking model. The second approach is a forward-looking, velocity-driven optimization strategy. The forward-looking model considers plant delays and empirical shift rate constraints on the CVT, as well as lag in the operation of the CVT due to shift characteristics. We implement an algorithm to minimize the effect of powertrain response lag in the system by considering powertrain loss and inertia loss due to CVT ratio changes.

There has been substantial research in recent years on model predictive control (MPC) for fuel economy optimization. MPC strategies have been developed for powertrains with CVTs [9, 10] and for hybrid electric vehicles with electronic-CVTs [11, 12]. Reference [13] implemented a nonlinear MPC strategy with OOL tracking for a hybrid electric vehicle. These MPC strategies focus on implementation of the optimal control, while the focus of the current paper is ratio schedule optimization and development of the P-OOL. The two approaches are not mutually exclusive; indeed, MPC could be implemented with P-OOL tracking in an approach similar to Ref. [13].

The primary contributions of this paper are: (1) development of the P-OOL, a new type of optimal operating line that includes combined engine and CVT efficiency, (2) a methodology for generating the P-OOL that is based on empirical data, which allows for faster and more accurate optimization than existing analytical approaches to CVT efficiency optimization, and (3) validation of the methodology using a backward-looking model to generate the P-OOL and a forward-looking model to verify results and implement a control strategy.

Results of the two optimization approaches indicate that P-OOL tracking has potential to improve overall powertrain fuel economy. Simulation results show that powertrain response lag is a key factor in enabling accurate P-OOL tracking. The proposed ratio scheduling and response lag control methodology is shown to improve vehicle fuel economy with real-time simulated driving conditions.

## Engine and CVT Efficiency

A 2.5L, 4-cylinder internal combustion engine was used for this analysis. The engine efficiency was measured at engine speed and crankshaft torque values within the range 600 - 6500 RPM and 20 - 260 Nm, respectively. These data were used to create an engine brake specific fuel consumption map, which determines the shape of the OOL. However, the data set contained multiple local maxima, which presents a concern for numerical optimization. In order to obtain a working surface that would not yield problems within the optimization routine, the empirical data was smoothed using a 4<sup>th</sup> order nonlinear regression surface polynomial fit. To avoid over-filtering, a combination of the initial data set and the filtered surface was developed using a torque-weighted relaxation. This alteration eliminated local maxima in the original dataset, while retaining realistic characteristics.

A three-dimensional representation of the final map is shown in Figure 2, where areas of red and blue indicate high and low engine efficiency, respectively.

CVT efficiency was determined using benchmarked CVT torque-loss data. The CVT efficiency is dependent on three variables: torque converter (TQC) turbine speed, CVT input torque, and CVT ratio. Torque-loss quantities were measured across ranges of torque converter turbine speeds and CVT input torque values. Additionally, this set of measurements was taken at five different CVT ratios. The ranges of TQC turbine speeds and CVT input torque values for this set of measurements were 0 - 6000 RPM and 0 - 250 Nm, respectively. The five CVT ratios at which these measurements were taken were 0.378, 0.7, 1.05, 1.5, and 2.631.

Due to the higher dimensionality of the set of input variables in the CVT efficiency data set, multiple linear regression was used to fit the data to a three-dimensional function. The predictors for the equation to determine CVT efficiency were formulated as functions of three variables: torque input to the
CVT, the TQC turbine speed, and the CVT ratio. The regression yields the coefficients that are combined with these predictors, resulting in a nonlinear function that can approximate the CVT efficiency at any point. This nonlinear function is of the form

$$\eta_{CVT} = b_1 + b_2 \frac{\omega_{eng}}{T_{eng}} + b_4 \log r_{CVT} + b_5 \frac{\omega_{eng}}{T_{eng}} +$$

(1)

where

$$r_{CVT} = \frac{\omega_{pri}}{\omega_{sec}}$$

Details of the curve-fitting process are available in [14]. Equation (1) shows that the CVT efficiency, $\eta_{CVT}$, is a function of the engine speed, engine torque, and the log of the CVT ratio. The coefficient values $b_i$ are constant terms determined from the regression fitting. The regression only needs to be performed once for the data set, and then this nonlinear function can predict the value of $\eta_{CVT}$ for any given point between the limits of the benchmark data set. Figure 3 shows regression surfaces generated using this methodology for three of the five CVT ratios, where areas of red and blue again indicate high and low CVT efficiency.

### Assumptions

Some system assumptions were made to limit the focus of this methodology. Torque converter losses are not considered for optimization. Although the torque converter is a vital component for power transferal within the vehicle powertrain, this assumption allows for an optimization problem with fewer constraints. The complexity of the fluid dynamics and mechanical losses of the torque converter -- and the associated simulation time requirements -- outweigh the accuracy gained from including them within the system.

Also, steady-state operating conditions were assumed, with constant engine coolant and transmission fluid temperature. In reality, it is known that engine and CVT efficiencies are dependent on these operating temperatures, but the effects are less significant than other factors considered. Fluctuation in temperature was neglected for this analysis.

### Backward-Looking Model Optimization

The backward-looking optimization methodology was developed to optimize the combined engine and CVT efficiency during power-on operation. From a vehicle cycle standpoint, this translates to only optimizing during positive acceleration. CVT efficiency is considered in the overall powertrain efficiency.

### Powertrain Model

For the case of a locked TQC, the primary CVT pulley speed is equal to the engine crankshaft speed, and CVT torque input is equal to engine crankshaft torque output:

$$\omega_{pri} = \omega_{eng}$$

(2)

$$T_{CVT_{in}} = T_{eng}$$

(3)
The methodology is initiated with a given forward vehicle velocity and commanded acceleration. From these parameters, the wheel power needed to meet the acceleration requirement is calculated using dynamic relationships, starting with the calculation of vehicle acceleration,

\[ a = \frac{d}{dt}(v), \]  

where \( v \) is forward vehicle velocity. The force applied to the wheels is

\[ F = ma \]

where \( m \) is the vehicle mass. The resistances of rolling friction, air, and grade make up the rest of the forces acting on the vehicle, and are shown in Equations (6)-(9):

\[ R_{rolling} = 0.02mg \cos \theta \]  
\[ R_{air} = 0.5 \rho_{air} C_d A v^2 \]  
\[ R_{grade} = mgsin \theta \]  
\[ F = R_{rolling} + R_{air} + R_{grade} \]

where \( g \) is the acceleration due to gravity, \( \theta \) is the grade of the driving surface, \( \rho_{air} \) is the density of air, \( C_d \) is a drag coefficient, and \( A \) is the frontal area of the vehicle. Equation 10 then gives the wheel power required to accelerate the vehicle,

\[ P_{wheel,demand} = (F_e + F_o) \cdot v \]  

From the demanded wheel power, the engine power requested is found using CVT efficiency,

\[ P_{eng, request} = \frac{P_{wheel,demand}}{\eta_{CVT}} \]  

Equation (11) is one of the constraints used for optimization. Two more constraints come from restricting the CVT ratio between a maximum and minimum predefined ratio:

\[ r_{min} \leq r \leq r_{max} \]  

The final constraint is a CVT shift rate limit. The CVT used in this analysis was assumed to shift from maximum to minimum ratio in one second. For the case of a time step of 0.1 seconds (as used in the optimization routine), the maximum ratio change was 10 percent of the previous value, as shown in Equations (13) and (14):

\[ r_{CVT} \leq r_{CVT, +} \frac{r_{max} - r_{min}}{10} \]  
\[ r_{CVT, -} \frac{r_{max} - r_{min}}{10} \leq r_{CVT} \]

Simple limits for locked and unlocked TQC cases were set to govern the optimization routine. When either the engine speed or the forward vehicle speed fell below a limiting value, the TQC was assumed to be slipping, or unlocked, and therefore the optimization was not performed because of lack of TQC efficiency consideration. The points that do not meet the criteria for optimization as outlined above correspond to points where only engine efficiency is considered. At these points, ratio limits and shift rate limits are still enacted on the system even though CVT efficiency is not being considered. This assures realistic CVT operation throughout the full schedule.

**Optimization Process**

The objective function in the optimization is an equation that relates fuel power to engine power through the engine speed and engine torque.

\[ \text{Minimize } P_{fuel} = \frac{\alpha_{eng} \cdot T_{eng}}{\eta_{eng}} \]  

By optimizing fuel power, minimal fuel should be consumed while maintaining sufficient engine power to meet the constraint given in Equation (11).

The backward-looking model optimization was performed using an iterative process. Vehicle velocity was specified by a drive test cycle (e.g. FTP-75), and Equations (4) - (11) were used to determine the requested engine power, and therefore, the engine speed and torque. Equation (15) was optimized using a standard numerical optimizer (MATLAB’s fmincon function).

As described above, the engine and CVT efficiency data were modified slightly to present more optimization-friendly functions. If any measurement inaccuracies exist in the empirical data, the alteration of inaccurate data could lead to inaccurate approximations. In optimization, inaccurate approximations can lead to false local maxima within the data, where an optimized solution may converge. To eliminate this possibility of local maximum convergence, an iterative procedure was developed for the optimization routine. Starting with the engine operation point on the OOL that coincides with the power requested by the engine, many random seed points were generated within a defined space around this starting point. Powertrain performance at each seed point was then optimized, and the solution that required the least fuel was taken as the optimal solution. Thus, at each time step, the numerical optimization process was repeated many times from a cloud of initial conditions (seed points), and the minimum-fuel solution was stored as the optimal result for the time step.

**Backward-Looking Optimization Results**

MATLAB was used to test the proposed methodology. The Bag-3, Bag-2, and Highway portions of the FTP-75 cycle were used as the cycle velocity input, and fuel economy was used as the sole comparator. Figure 4 shows the entire cycle; note that the Bag-1 and Bag-3 portions are identical, if temperature conditions are neglected, so Bag-1 was not used. The results from each portion display very similar characteristics. Only the Bag-3 results are discussed in detail here. Fuel economy for each cycle is also presented.

Optimized solutions from the multiple-seed-point optimization approach are shown in Figure 5. The optimized solutions are plotted point by point on the engine speed versus engine torque plane. The solid blue line represents the OOL, cyan blue points represent solutions that were not optimized
In these situations, CVT efficiency played the dominant role in the overall powertrain efficiency calculation.

The second characteristic of interest is in the region of 1800-2500 RPM. The majority of optimal solutions in this region fall below the original (blue) OOL. This is believed to occur because in that region, the CVT efficiency is a significant factor in overall powertrain efficiency, which justifies deviating away from an area of higher engine efficiency.

Figure 6 shows the CVT ratio schedule corresponding to the optimization results in Figure 5. In this simulation, the CVT shift rate over each time step was limited. This limitation prevented rapid changes in the CVT ratio. As shown in Figure 6, the maximum CVT ratio was implemented five times during the Bag-3 simulation, while most of the cycle was completed at a lower CVT ratio.

Figure 7 displays the calculated powertrain efficiency during a 25-second portion of the Bag-3 cycle for both the unlocked TQC points with OOL tracking, and red points represent solutions that were optimized.

Two interesting characteristics are visible in Figure 5. First, for engine speeds less than 1200 RPM, many optimized (red) solutions deviate away from the OOL towards a region of high torque and low engine speed. This phenomenon is due to low acceleration (and therefore low wheel power demand), as well as the shape of the CVT efficiency contours. As shown in Figure 3, areas of high torque and low engine speed exhibit higher CVT efficiency, thus the optimal solutions tend to deviate in that direction when the power request is not a limiting factor. Similarly, the vertical line of red solutions located near an engine speed of roughly 750 RPM indicates that those solutions, which met the criteria for optimization with a very low power request, deviated towards the highest CVT efficiency before meeting the engine idle speed constraint.
OOL tracking (without optimization) and optimized ratio methods. Fuel economy improvement can be seen in the optimized case.

The backward-looking model yields different overall fuel economy results between the optimized and non-optimized solutions. The percent improvement in fuel economy for both scenarios of all three portions of the FTP75 cycle are displayed in Table 1.

### Forward-Looking Model Optimization

Following the backward-looking model optimization and the determination of the optimized points in Figure 5, a forward-looking model was developed to simulate real-time CVT ratio management. Figure 8 shows the general outline of the forward-looking model, which was implemented in MATLAB Simulink. Portions of the FTP-75 cycle were used as the desired speed input for the model.

The environment block contains the external resistance forces on the vehicle. The formulae used for aerodynamic, rolling, and grade resistances are given by Equations (6) - (9). These resistances are considered in the control and plant model.

The controller block contains the required power calculation, engine RPM and engine torque lookup according to required power, CVT ratio calculation, and the shift rate constraint for the CVT ratio. As this model is velocity-driven, the required power is calculated in the controller block using the actual vehicle speed along with the desired speed and the environmental resistances. A PID controller is used to evaluate the power required to eliminate the difference between the actual and desired speed. The required power is calculated using Equation (16),

$$P = \frac{(v_d \alpha + v_d F_i + P_{PID})}{\eta_{CVT}}$$

where $v_d$ is the required speed, $\alpha$ is the required acceleration, and $\eta_{CVT}$ is determined using the data-based nonlinear function from Equation (1). Braking force is applied when the required power is negative.

The plant model consists of a simplified, physics-based engine model, torque converter model, CVT model, vehicle model, and fuel consumption and CVT efficiency calculations. The equations of the shift rate constraint applied in the forward-looking model are derived from calibrated data from a representative vehicle. The shift rate limits for upshift and downshift are a function of CVT ratio, as shown in Figure 9.

### P-OOL Implementation

The optimized points in Figure 5 were obtained as a result of an offline optimization process, in which each commanded velocity change required optimization with many seed points. This process produces a high computational load and a long processing time; it cannot be applied in a real-time vehicle as a part of online optimization. To implement these results in a real-time driving situation, a curve fit was generated for the optimized data points, as shown in Figure 10.

By combining the curve fit and OOL, we create the P-OOL, which includes both CVT and engine efficiency. Outlying points and unlocked torque converter operating points (engine speed less than 1000 rpm) were ignored while performing the curve fit calculations. The resulting curve is a 4th order polynomial.

The backward-looking cycle-driven model was first used to test the fuel economy of a P-OOL tracking strategy. It was found that the vehicle achieved maximum fuel economy when a central region of the OOL was replaced with the P-OOL, as shown in Figure 11.

For the Bag-3 cycle, the P-OOL shown in Figure 11 resulted in a 0.56% improvement in fuel economy, compared to OOL tracking alone. This is less than the 1.28% improvement achieved using the pointwise-optimization approach, but it is achieved with a significantly reduced computational load that could be implemented in real time.

When the forward-looking model was used with P-OOL tracking, it was found that there was no significant improvement in fuel economy compared to OOL tracking. The primary
reasons for these results are the model shift characteristics, plant delays, and the noise factor present in the model. In a realistic vehicle simulation, the operating points deviate away from the P-OOL significantly. At any time step, the actual operating point may differ from the target operating point by up to 30 Nm. The magnitude of these errors is significantly higher than the deivation of the P-OOL from the OOL, such that there is no measurable improvement between the two strategies. To correct these operating point errors, a powertrain response lag compensation algorithm was developed.

Powertrain Response Lag Compensation

Figure 12 displays the locations of the engine operating point for an increase in vehicle speed from point A to point B. With powertrain response lag (PRL) in the system, the system follows path A-C-B; error is induced in the vehicle actual speed, which results in a higher CVT ratio, causing the operating point to shift to point C before reaching point B. Without any powertrain response lag in the system, the vehicle would accelerate to point B directly from point A with constant engine speed. This would ensure accurate tracking of an optimal operating line (OOL or P-OOL), resulting in improved fuel economy.

To address the tracking errors and regain the benefits of P-OOL implementation, a PRL compensation algorithm was implemented. This algorithm, based on the method proposed in Ref. [7], focuses on predicting the desired vehicle speed after a time delay equivalent to the PRL. The future engine speed is predicted using a linear equation,

\[
\omega^* = \omega + \alpha \tau
\]

where \(\omega^*\) is the predicted output shaft speed, \(\omega\) is the actual output shaft speed, \(\alpha\) is the output shaft acceleration, and \(\tau\) is the assumed PRL time constant.

The CVT ratio \(r_{CVT}\) and predicted CVT ratio \(r_{CVT}^*\) are determined from \(\omega\) and \(\omega^*\)

\[
r_{CVT} = \frac{\text{Desired engine RPM}}{\omega}
\]

\[
r_{CVT}^* = \frac{\text{Desired engine RPM}}{\omega^*}
\]

In the forward-looking model, the predicted CVT ratio is then utilized in the plant model to calculate the torque delivered to the output shaft.

P-OOL Tracking and Lag Compensation Results

Simulations were performed on the Bag-3 portion of the FTP-75 cycle with the PRL control algorithm. The PID controller was tuned to maintain the vehicle actual speed within 1 mph of the desired vehicle speed throughout the simulation. Results of a representative four-second interval are shown in Figures 13-15. Figure 13 shows the PRL control opting for a lower CVT ratio than the PID controller without
The PRL compensation algorithm with P-OOL tracking yielded a fuel economy improvement of 0.40% compared to simple OOL tracking in the forward-looking model.

Conclusions

A ratio management and optimization strategy was developed for a powertrain consisting of an internal combustion engine and a CVT. The strategy considers the effects of variable CVT efficiency in addition to engine efficiency. Empirical data were used to generate models of engine and CVT efficiency over a range of operating conditions, and then a backward-looking, cycle-driven model was used in an iterative optimization process to determine the optimal operating points. This approach showed significant gains in fuel economy, assuming perfect tracking of a given speed profile. It is evident that with this methodology, lower fuel consumption rates can be obtained when the engine operating point is allowed to deviate from the OOL if a higher CVT efficiency is attainable.

Next, a more realistic model was used. The velocity-driven methodology presented in the forward-looking model resembles real-time vehicle operation more closely than the backward-looking strategy. A more realistic CVT shift rate constraint was applied in the model using limit equations developed from empirical test data. A predictive algorithm was used to compensate for powertrain response lag. The forward-looking model was simulated using MATLAB and Simulink for various portions of the FTP-75 cycle inputs. Simulation results show improvements in vehicle fuel economy.

We find that CVT efficiency is a significant factor in overall powertrain efficiency. Neglecting the effects of CVT efficiency and tracking the engine OOL alone results in a sacrifice of fuel economy. These results also underscore the importance of including vehicle shift dynamics in any strategy to track the powertrain-optimal operating line. Finally, these results reinforce the importance of future work in developing a control strategy to further shorten the gap between the desired powertrain operating points and the actual parameter values. While the powertrain response lag compensation strategy used in this paper is a simple and effective strategy for implementing P-OOL tracking, other control strategies, such as MPC, may enable further fuel economy improvement.

References


FIGURE 13 CVT ratio comparison.

FIGURE 14 Fuel rate comparison.

FIGURE 15 Speed trace comparison.
CVT RATIO SCHEDULING OPTIMIZATION WITH CONSIDERATION OF ENGINE AND TRANSMISSION EFFICIENCY


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